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TRAVELING-WAVE PARAMETRIC AMPLIFIER USING THIN FILM

by

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A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

In Charge of Major Work

Head of Major Department

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1962

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INTRODUCTION

Amplification of a weak signal to a high enough level that information can be obtained is an important subject. Energy should be supplied to the signal without distorting it. In vacuum tube and transistor amplifiers, d-c energy is used. Energy, therefore, is converted from one frequency to another by means of the tube or the transistor.

Energy can be converted to different forms using a time-varying element in a circuit. In the last decade, circuits with one or more variable parameters have been used in amplifiers and frequency converters. Energy is stored in a parameter, such as a capacitive or inductive reactance, and delivered to the weak signal at the right time giving the signal a boost. The energy used is not of a d-c nature; rather, it is of a high-frequency nature of about the same order of magnitude as that of the signal. The high-frequency energy known as the pump is transferred through the variable parameter to strengthen the signal, and thus gain occurs. Manley and Rowe (1, 2) have proved that when two periodic signals at frequencies f_1 and f_2 are impressed across an element, the value of which varies periodically, new frequencies are generated. The values of these frequencies are formed by sums and differences of the harmonics of the original signals. Any specific signal can be obtained by using filters. In their paper, Manley and Rowe (1) have derived the two famous equations relating the average powers at different frequencies in a variable reactance which contains no hysteresis. The two independent equations are:

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{m,n}}{mf_1 + nf_2} = 0$$

$$\sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{nP_{m,n}}{mf_1 + nf_2} = 0$$

(1)

where $P_{m,n}$ is the average power delivered to the reactance at a frequency of $mf_1 + nf_2$. These general equations give the information about gain, modulation, frequency converting and so forth.

Parametric amplifiers have gained special interest theoretically and experimentally among engineers. Heffner and Wade (3) and Heffner and Kotzebue (4) and many others investigated and succeeded in proving the feasibility of making such devices. They used back-biased semiconductor diodes which act as time-variable capacitors. More recently Pohm and Read (5) have succeeded in using ferromagnetic thin film as the time-variable parameter in parametric amplifiers. Thin films also have been used in other parametric devices such as the balanced modulator (6) and the para-metron (7).

Another version of amplifiers using variable parameters is the traveling-wave parametric amplifiers. As the name implies, the wave is amplified while it travels. Normally a traveling wave along a guide is attenuated. Any waveguide whose walls have finite conductivity has inherent losses built in it. If time- and space-variable parameters are introduced periodically or distributively along the guide to deliver a booster energy to the signal as it passes by, losses can be overcome and net gain is obtained. A simple, but not exact, way to visualize this mechanism is the telephone

line with repeaters placed periodically across it. The traveling wave amplifier has an outstanding salient feature of having a wide bandwidth.

This idea of amplification intrigued many investigators (8 - 12), especially when semiconductor diodes became available for high-frequency operation. The investigators cited in the references have analyzed the problem from different points of view. In their work the back-biased diodes were placed across the signal line. The diodes were pumped in the right phase relationship with the signal, thus transferring the pump energy to the signal, and gain was observed. Tien and Suhl (12) have embedded the two propagating structures in a ferromagnetic medium. The variable parameter here is of distributed nature and it varies in time and space. Such a parameter gives rise to time- and space-variable inductive reactance which is a requirement for parametric amplification in a traveling-wave structure.

In this paper a traveling-wave parametric amplifier using ferromagnetic thin film will be analyzed from a circuit point of view. The thin film will be considered to be distributed along the line, and the line will be considered as small sections in cascade. The length of each section is also considered to be small with respect to a wavelength. Each section therefore can be represented by a two-port network. By such an approach the propagation constant is found to give a negative attenuation constant which results in gain. Also, a relationship is found which must be satisfied in order for the propagation to take place, otherwise there will be attenuation. From this relationship the bandwidth or the passband of the amplifier is found. The pump propagating structure is a helix

wound around the signal line with such a pitch that the two waves travel with the same velocity and maintain their proper phase relationship.

In order to keep the analysis from getting very complicated, the propagating structures were assumed to be lossless. Additional restrictions will be discussed when they arise. It will be shown that the results of this analysis satisfy the Manley-Rowe general energy equations, Equations 1, a requirement that must be met in any parametric device.

FERROMAGNETIC THIN FILM

The film used in this project was made by depositing nickel-iron permalloy on a heated clean glass substrate, inside an evacuated chamber in the presence of an external d-c magnetic field. The permalloy contained 80 percent nickel and 20 percent iron. The external magnetic field directs the evaporated permalloy particles such that, when they are deposited on the substrate, their magnetic dipoles are pointed in one direction parallel to the applied field. The film therefore possesses a uni-axial anisotropy; its magnetization has a preferred direction parallel to the applied field. This direction is known as "easy" or "rest" direction. Perpendicular to the easy direction in the plane of the film is the "hard" direction. The film forms a single region, commonly called domain, with a net magnetization vector $\overline{M}$ pointing in one direction, namely the easy direction. It is possible to cause the film to break up into multiple domains by applying a field in the hard direction then turning the field off. To keep this from happening, a d-c bias field is applied. For more information about domain theory reference (13) is very helpful.

To understand how the thin film works, the following simple example is used. Let two orthogonal windings be wound around a glass substrate with a magnetic film deposited on it as shown in Figure 1. The magnetic field set up by the pump current i_p is parallel to the magnetization vector $\overline{M}$ of the film, while the field set up by the signal current i_s tends to rotate the vector $\overline{M}$ into the hard direction. A bias current I_{d-c} is added to keep the film from breaking into multiple domains. The pump field in the easy direction fluctuates about the bias field. The interaction

between the two fields set up by i_p and i_s will cause $\overline{M}$ to vibrate about its equilibrium position. This vibration will induce a voltage in the winding of the signal circuit, and this voltage is coupled out. Thus energy is transferred from the pump circuit to the signal circuit, which are mutually perpendicular, by means of the magnetization vector of thin film.

The motion of the magnetization vector will be analyzed mathematically. A coordinate system is chosen such that the rest direction is along the x-axis as shown in Figure 2. The thin film is in the x-y plane. When a transverse field is applied, the vector $\overline{M}$ swings through an angle of ϕ in the plane of the film, and through a small angle of ψ out of that plane. The angle ψ can be visualized by giving a compass needle a small horizontal kick while in equilibrium position. The needle swings back, forth, up and down. The tip follows an ellipsoidal path. Landau and Lifshitz have described the motion of the magnetization vector by an equation known by their name and modified by Gilbert (14). Gillete and Oshima (15) have solved the equation to describe the two angles ϕ and ψ as functions of time. The modified Landau-Lifshitz equation is

$$\dot{\overline{M}} = \gamma \ \overline{M} \times \overline{H} - \frac{\alpha}{M} \ (\overline{M} \times \dot{\overline{M}}) \tag{2}$$

where

$\overline{H}$ is the effective magnetic field; more precisely,

$$\overline{H} = \overline{H}_a + \overline{H}_k + \overline{H}_d$$

$\overline{H}_a$ = the externally applied field

$\overline{H}_k$ = the anisotropy field which tries to turn the vector

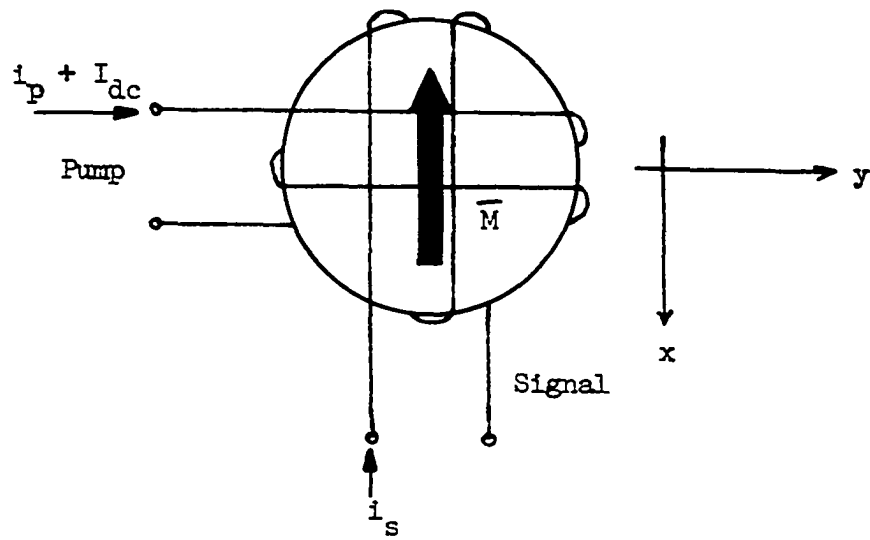$\overline{M}$ to its equilibrium position

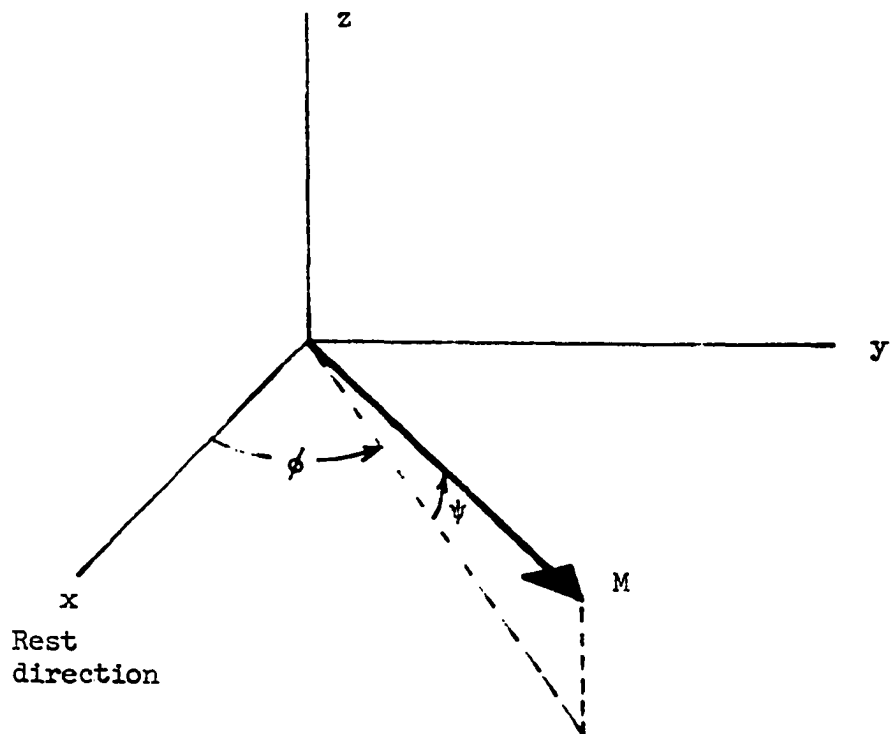Figure 1. A simple thin film circuit

Figure 2. Coordinate system for thin film
inductance analysis

$\overline{H}_d$ = the demagnetizing field which arises from the out-of-plane motion of the vector $\overline{M}$ through angle ψ .

α is the phenomenological damping factor. It acts like resistance in electric circuits or friction in motion.

γ is the gyromagnetic ratio.

Rewriting Equation 2,

$$\dot{\overline{M}} = \gamma \ \overline{M} \times (\overline{H} - \frac{\alpha}{\gamma M} \ [\gamma \overline{M} \times (\overline{H} - \frac{\alpha}{\gamma M} \ \dot{\overline{M}})])$$

After some manipulation, the following is obtained:

$$\dot{\overline{M}} = \frac{\gamma}{1 + \alpha^2} \ \overline{M} \times \overline{H} - \frac{\alpha \gamma}{(1 + \alpha^2)M} \ \overline{M} \times \overline{M} \times \overline{H} \tag{3}$$

Now, dividing by M^2 and letting

$$\tau = \frac{\gamma}{1 + \alpha^2} M t$$

$$d\tau = \frac{\gamma}{1 + \alpha^2} M \ dt$$

Equation 3 becomes

$$\frac{d}{d\tau} (\frac{\overline{M}}{M}) = -(\frac{\overline{M}}{M} \times \frac{\overline{H}}{M}) - \alpha[\frac{\overline{M}}{M} \times (\frac{\overline{M}}{M} \times \frac{\overline{H}}{M})] \tag{4}$$

To break this vector equation into three scalar equations, let

$$\frac{\overline{M}}{M} = m_x \ \overline{i} + m_y \ \overline{j} + m_z \ \overline{k} \tag{5}$$

$$\frac{\overline{H}_a}{M} = h_x \ \overline{i} + h_y \ \overline{j} + h_z \ \overline{k} \tag{6}$$

$$\frac{\overline{H}_k + \overline{H}_d}{M} = -(k_x + d_x)m_x\,\overline{i} - (k_y + d_y)m_y\,\overline{j} - (k_z + d_z)m_z\,\overline{k}$$

$$= -n_x m_x\,\overline{i} - n_y m_y\,\overline{j} - n_z m_z\,\overline{k} \qquad (7)$$

From Figure 2,

$$M_x = \cos\psi\cos\phi \qquad (8)$$

$$M_y = \cos\psi\sin\phi \qquad (9)$$

$$M_z = \sin\psi \qquad (10)$$

Also, for ferromagnetic thin film

$$d_x = d_y = 0;\quad d_z = \frac{1}{\mu_o} \qquad (11)$$

$$k_x = 0;\quad k_y = k_z = a \qquad (12)$$

where

$$a = 2\frac{K_1}{M^2}$$

and K_1 is the coefficient in the anisotropy energy equation derived by Kittel (13).

Putting Equations 5 through 12 in 4 the following equations are obtained:

$$\frac{d\phi}{d\tau} = [-\frac{\psi}{\mu_o} + h_z] - \alpha[a\sin\phi\cos\phi + h_x\sin\phi - h_y\cos\phi] \qquad (13)$$

$$\frac{d}{d\tau} = [a\sin\phi\cos\phi + h_x\sin\phi - h_y\cos\phi] + \alpha[\frac{-\psi}{\mu_o} + h_z] \qquad (14)$$

For small vibrations the angles are small such that $\sin\phi \approx \phi$, $\cos\phi \approx 1$, and ψ is small. There is no magnetic field applied in the z direction, so $h_z = 0$. Equations 13 and 14 then become

$$\frac{d\phi}{d\tau} = \frac{\psi}{\mu_o} - \alpha \left[a\,\phi + h_x\,\phi - h_y \right] \tag{15}$$

$$\frac{d\psi}{d\tau} = a\,\phi + h_x\,\phi - h_y - \frac{\alpha\,\psi}{\mu_o} \tag{16}$$

Next, differentiating Equation 15 with respect to τ leads to

$$\frac{d^2\phi}{d\tau^2} = -\frac{1}{\mu_o}\frac{d\psi}{d\tau} - \alpha\frac{d}{d\tau}\left[a\,\phi + h_x\,\phi - h_y \right] \tag{17}$$

The last term on the right hand side represents the torque in the ϕ direction, which varies slowly. Also α is a small number. The whole term therefore can be neglected.

Substituting Equation 16 in Equation 17 gives the following results:

$$\frac{d^2\phi}{d\tau^2} = -\frac{1}{\mu_o}(a + h_x)\,\phi + \frac{1}{\mu_o}\,h_y + \frac{\alpha\,\psi}{\mu_o^2} \tag{18}$$

Solving for ψ from Equation 15 and substituting in Equation 18 yields

$$\mu_o \frac{d^2\phi}{d\tau^2} + \alpha\frac{d\phi}{d\tau} + \left[a(1 + \alpha^2) + h_x(1 + \alpha^2) \right]\phi = h_y(1 + \alpha^2)$$

where $\alpha \approx 0.015$ for 80-20 permalloy. Therefore, neglecting second order terms in α, the differential equation in ϕ reduces to

$$\mu_o \frac{d^2\phi}{d\tau^2} + \alpha\frac{d\phi}{d\tau} + \left[a + h_x \right]\phi = h_y$$

Also $d\tau = \gamma M\,dt$ and therefore

$$\frac{\mu_o}{\gamma^2 M^2} \frac{d^2\phi}{dt^2} + \frac{\alpha}{\gamma M} \frac{d\phi}{dt} + [a + h_x] \phi = h_y \qquad (19)$$

It should be noticed that $a = \frac{2K_1}{M^2}$, and h_x and h_y are normalized with respect to M. Therefore Equation 19 should be multiplied by M, and the result is

$$\frac{\mu_o}{\gamma^2 M} \phi + \frac{\alpha}{\gamma} \phi + [\frac{2K_1}{M} + H_x] \phi = H_y \qquad (20)$$

where

$$\frac{2K_1}{M} = H_k, \text{ the anisotropy field}$$

and

$$H_x = H_b + h_p(t)$$

where H_b is the d-c field and $h_p(t)$ is the a-c field which is a function of time. H_y will now be referred to as H_T, and Equation 20 becomes

$$\frac{\mu_o}{\gamma^2 M} \phi + \frac{\alpha}{\gamma} \phi + [H_k + H_b + h_p] = H_T \qquad (21)$$

Looking at Figures 1 and 2 it can be seen that the component of M which induces voltage in the signal winding is M sin ϕ. If ϕ is assumed to be small, this component is simply M ϕ where ϕ is a time-varying function given by Equation 21.

The total flux density B is given by the following equation

$$B = \mu_o H + M \phi , \quad M >> \mu_o H$$

The total flux can then be approximated by $A_m M \phi$, where A_m = cross sectional area of the film. Thus, the flux linkage $\lambda = A_m M \phi = \lambda_m \phi$ for

M = 1. Solving for ϕ leads to

$$\phi = \frac{\lambda}{\lambda_m}$$

and substituting this in Equation 21 yields

$$\frac{\mu_o}{\gamma^2 M \lambda_m} \ddot{\lambda} + \frac{\alpha}{\gamma \lambda_m} \dot{\lambda} + (H_k + H_b + h_p) \frac{\lambda}{\lambda_m} = H_T \tag{22}$$

H_T is the magnetic field in the transverse direction and is proportional to the current i_s exciting it. Therefore,

$$H_T = k\, i_s$$

where k is a proportionality constant. Substituting this in Equation 22 leads to

$$\frac{\mu_o}{\gamma^2 M k \lambda_m} \ddot{\lambda} + \frac{\alpha}{\gamma \lambda_m k} \dot{\lambda} + \frac{H_k + H_b}{k \lambda_m} \left(1 + \frac{h_p}{H_k + H_b}\right) \lambda = i_s \tag{23}$$

Since $\dot{\lambda} = v$, the voltage induced, Equation 23 can be written in terms of voltage as

$$\frac{\mu_o}{\gamma^2 M k \lambda_m} \frac{dv}{dt} + \frac{\alpha}{\gamma \lambda_m k} v + \int \frac{H_k + H_b}{k \lambda_m} \left(1 + \frac{h_p}{H_k + H_b}\right) v\, dt = i_s \tag{24}$$

By comparing this with

$$C \frac{dv}{dt} + G v + \int \frac{1}{L} v\, dt = i$$

it can be seen that Equation 24 represents a situation that is analogous to a circuit with C, G and L in parallel and driven by a current source of value i_s.

The analogous parameters are

$$C = \frac{\mu_o}{\gamma^2 \, M \, k \, \lambda_m}$$

$$G = \frac{\alpha}{\gamma \, \lambda_m \, k}$$

$$\hat{L} = L_o \left(\frac{1}{1 + \dfrac{h_p(t)}{H_k + H_b}} \right)$$

(25)

where

$$L_o = \frac{k \, \lambda_m}{H_k + H_b}$$

The undamped resonant frequency can now be found from the circuit parameters. It is

$$\omega = \sqrt{\frac{1}{LC}} \approx \frac{1}{\sqrt{L_o C}}$$

This is a good approximation because h_p is small with respect to $H_k + H_b$. Therefore,

$$\omega \approx \left[\frac{\gamma^2 \, M \, (H_k + H_b)}{\mu_o} \right]^{1/2}$$

(26)

This is the ferromagnetic resonance (16) which is the same as the electron-spin resonance. At this frequency all transverse field energy is absorbed and lost as heat.

For 80-20 permalloy, the following approximate constants may be used:

$$H_k = 200 \text{ Amp-turns per meter}$$

$$\gamma = 2.21 \times 10^5 \frac{cps}{Amp\text{-}turns \ per \ meter}$$

M = 1 weber per meter square

μ_o = 4π x 10^{-7} henry per meter.

If H_b of 200 Amp-turns per meter is applied, the resonant frequency is 630 megacycles per second. The resonant frequency increases with H_b as shown in Equation 26.

The three parameters in Equation 25 can now be evaluated using the above information:

$$C = \frac{2.57 \times 10^{-17}}{\lambda_m k}$$

$$G = \frac{6.78 \times 10^{-8}}{\lambda_m k} \quad (\text{or } R = 1.47 \times 10^7 \lambda_m k)$$

$$L_o = 2.5 \times 10^{-3} \lambda_m k$$

Suppose that the signal frequency is 100 MC, which is much below the resonant frequency. Then

$$\frac{1}{\omega C} = 6.2 \times 10^7 \lambda_m k$$

$$R = 1.47 \times 10^7 \lambda_m k$$

$$\omega L = 1.57 \times 10^6 \lambda_m k$$

Therefore, at frequencies much below resonance, the circuit is primarily inductive.

Going back to Equation 25 and letting

$$h_p(t) = H_p \cos \omega_p t$$

the time-varying inductance becomes

$$\hat{L} = L_0 \left(\cfrac{1}{1 + \cfrac{H_p \cos \omega_p t}{H_k + H_b}} \right)$$

Next, let

$$\frac{H_p}{H_k + H_b} = a \qquad\qquad (27)$$

Then

$$\hat{L} = L_0 \left(\frac{1}{1 + a \cos \omega_p t} \right)$$

where a is a small number in the case being studied.

The above fraction can be expanded in series form as

$$\hat{L} = L_0 \left[1 - a \cos \omega_p t + (a \cos \omega_p t)^2 + \cdots \right]$$

and, to a good approximation for small a,

$$\hat{L} \approx L_0 (1 - a \cos \omega_p t) \qquad\qquad (28)$$

Equation 28 shows that inductance $\hat{L}$ is a time-varying function. This suggests that this element could be utilized in a circuit to give parametric amplification, frequency mixing, modulation or other phenomena which must satisfy the Manley-Rowe general energy equations, Equation 1.

It will be demonstrated in the following sections how the negative resistance arises due to frequency mixing and how this phenomenon is used in parametric amplification.

DERIVATION OF NEGATIVE RESISTANCE

At frequencies below the ferromagnetic resonance, Equation 28 describes the inductance of the circuit. Looking from the terminals of the signal circuit in Figure 1, the inductance is given by

$$\hat{L} = L_o - L_y \cos \omega_p t$$

where

$$L_y = a \, L_o$$

When two signals of different frequencies f_1 and f_2 are impressed across the terminals of the signal circuit the input characteristics of the signal circuit will have inductive reactance and may also have negative resistance under certain conditions. This will be shown to be true if the two frequencies are related to the pump frequency by

$$\omega_1 + \omega_2 = \omega_p \qquad (29)$$

Let two currents i_1 and i_2 flow in the signal terminals where

$$i_1 = I_1 \cos (\omega_1 t + \theta_1)$$

$$i_2 = I_2 \cos (\omega_2 t + \theta_2)$$

The phase angles θ_1 and θ_2 are taken with reference to the pump current

$$i_p = I_p \cos \omega_p t$$

The induced voltage across the terminals is then

$$e = \frac{d}{dt} (\hat{L} \, i)$$

where·

$$i = i_1 + i_2$$

Performing the indicated differentiation leads to

$$e = [L_o - L_y \cos \omega_p t][-\omega_1 I_1 \sin(\omega_1 t + \theta_1) - \omega_2 I_2 \sin(\omega_2 t + \theta_2)]$$

$$+ L_y \omega_p \sin \omega_p t [I_1 \cos(\omega_1 t + \theta_1) + I_2 \cos(\omega_2 t + \theta_2)]$$

$$= -L_o[\omega_1 I_1 \sin(\omega_1 t + \theta_1) + \omega_2 I_2 \sin(\omega_2 t + \theta)]$$

$$+ \frac{1}{2} L_y \omega_1 I_1 [\sin(\omega_p t + \omega_1 t + \theta_1) - \sin(\omega_p t - \omega_1 t - \theta_1)]$$

$$+ \frac{1}{2} L_y \omega_2 I_2 [\sin(\omega_p t + \omega_2 t + \theta_2) - \sin(\omega_p t - \omega_2 t - \theta_2)]$$

$$+ \frac{1}{2} L_y \omega_p I_1 [\sin(\omega_p t + \omega_1 t + \theta_1) + \sin(\omega_p t - \omega_1 t - \theta_1)]$$

$$+ \frac{1}{2} L_y \omega_p I_2 [\sin(\omega_p t + \omega_2 t + \theta_2) + \sin(\omega_p t - \omega_2 t - \theta_2)] \tag{30}$$

Next it will be assumed that only ω_1, ω_2 and ω_p are allowed to exist by proper filtering. Applying this assumption to Equation 30 the following is readily obtained:

$$e = -L_o[\omega_1 I_1 \sin(\omega_1 t + \theta_1) + \omega_2 I_2 \sin(\omega_2 t + \theta_2)]$$

$$- \frac{1}{2} L_y \omega_2 I_2 \sin(\omega_1 t - \theta_2) - \frac{1}{2} L_y \omega_1 I_1 \sin(\omega_2 t - \theta_1)$$

$$+ \frac{1}{2} L_y \omega_p I_2 \sin(\omega_1 t - \theta_2) + \frac{1}{2} L_y \omega_p I_1 \sin(\omega_2 t - \theta_1)$$

This equation represents the sum of two induced voltages at two different frequencies. The two voltages can be written explicitly as follows, if Equation 29 is applied:

$$e_1 = \frac{1}{2} L_y \omega_1 I_2 \sin(\omega_1 t - \theta_2) - L_o \omega_1 I_1 \sin(\omega_1 t + \theta_1) \qquad (31)$$

$$e_2 = \frac{1}{2} L_y \omega_2 I_1 \sin(\omega_2 t - \theta_1) - L_o \omega_2 I_2 \sin(\omega_2 t + \theta_2) \qquad (32)$$

The impedance at ω_1 is found easily now as the ratio of voltage to current. In phasor notation, letting $\sin \omega t$ transform into e^{j0},

$$\hat{Z}_1 = \frac{\frac{1}{2} L_y \omega_1 I_2 e^{-j\theta_2} - L_o \omega_1 I_1 e^{j\theta_1}}{I_1 e^{j(\theta_1 + \pi/2)}}$$

$$= -j \frac{1}{2} L_y \omega_1 \frac{I_2}{I_1} e^{-j(\theta_1 + \theta_2)} + j L_o \omega_1$$

$$= -\frac{1}{2} L_y \omega_1 \frac{I_2}{I_1} \sin(\theta_1 + \theta_2) + j[L_o \omega_1 - \frac{1}{2} L_y \omega_1 \frac{I_2}{I_1} \cos(\theta_1 + \theta_2)]$$

Therefore, the real and imaginary parts of $\hat{Z}_1$ are

$$\hat{R}_1 = -\frac{1}{2} L_y \omega_1 \frac{I_2}{I_1} \sin(\theta_1 + \theta_2)$$

$$\hat{X}_1 = L_o \omega_1 - \frac{1}{2} L_y \omega_1 \frac{I_2}{I_1} \cos(\theta_1 + \theta_2)$$

Similarly,

$$\hat{R}_2 = -\frac{1}{2} L_y \omega_2 \frac{I_1}{I_2} \sin(\theta_1 + \theta_2)$$

$$\hat{X}_2 = L_o \omega_2 - \frac{1}{2} L_y \omega_2 \frac{I_1}{I_2} \cos(\theta_1 + \theta_2)$$

The maximum negative values of the resistances occur when the phase angles are chosen such that

$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

Therefore, for this condition

$$\hat{R}_1 = -\frac{1}{2} L_y \omega_1 \frac{I_2}{I_1}$$

$$\hat{R}_2 = -\frac{1}{2} L_y \omega_2 \frac{I_1}{I_2}$$

(33)

and

$$\hat{X}_1 = L_o \omega_1$$

$$\hat{X}_2 = L_o \omega_2$$

(34)

This shows that when two signals at different frequencies are impressed across a time-varying inductance where the three frequencies are related as in Equation 29, the inductance exhibits a negative resistance component of impedance, a phenomenon that may be used for amplification.

In parametric amplifiers the signal and the pump are the only two frequencies applied from external sources. The third frequency is generated by the mixing action. This was predicted by Manley and Rowe (1). In Figure 1, the magnetization vector $\overline{M}$ vibrates with an infinite number of frequencies, the values of which are the all possible sums and differences of f_s and f_p and their harmonics. By proper tuning or filtering a signal of any desired frequency can be coupled out. The two frequencies f_s and $f_p - f_s$ are of particular interest. The frequency $f_p - f_s$ is often called the idler frequency f_i. A better name might be the intermediate frequency. The energy at f_p is converted to energy at the idler frequency f_i through the time-varying inductance which varies with a frequency of f_p. Again, the energy of f_i, through the mixing action in the inductor, is converted to

energy at frequency of $f_p - f_i$. This frequency is just f_s and hence gain at the signal frequency may exist. The idler frequency therefore serves as intermediate step between f_p and f_s. Equation 29 may be written in terms of signal and idler frequencies as

$$\omega_s + \omega_i = \omega_p \tag{35}$$

ANALYSIS OF THE TRAVELING-WAVE AMPLIFIER

In the previous discussion, it has shown that when two signals at different frequencies are impressed across a time-varying inductance, negative resistance can be obtained. Power gain can be achieved in a circuit with negative resistance and this will be used in the amplifier discussed in this thesis.

In the problem at hand, the most convenient guide for the traveling wave at frequencies below the ferromagnetic resonance is a transmission line of any form. The choice of form of the line is a matter of convenience in constructing the amplifier. The negative resistance incorporated with the transmission line is a basic component of the traveling-wave parametric amplifier. The word "parametric" is used because the amplification occurs due to the variation of one of the parameters in the circuit.

A traveling-wave is described in steady state by a voltage or a current wave as functions of the phase shift constant β and by the positive (or negative) attenuation constant α. In this analysis, therefore, the voltage and the currents expressions will be developed. From these, power gain can easily be found. To prove the correctness of the results, Manley-Rowe general energy equations will be shown to be satisfied.

The analysis will be carried on from circuit theory point of view. A transmission line can be considered as a two-port, symmetrical network, the elements of which are found from the input impedance of transmission line. For a line of characteristic impedance of R_o and terminated in a load of Z_L, the input impedance is given by

$$Z_{in} = R_o \frac{Z_L \cos \beta d + j R_o \sin \beta_d}{R_o \cos \beta d + j Z_L \sin \beta_d} \qquad (36)$$

where β is the phase shift constant and d is the length of the line. The equivalent circuit representation of a transmission line is given in Figure 3. The series and shunt parameters can now be found by equating the input impedances of the two circuits shown in Figure 3 for the two different terminating impedances. Two convenient values of Z_L are open and short circuits. The values of Z_a and Z_b are found as

$$Z_a = j R_o \tan \frac{\beta d}{2}$$

$$Z_b = -j R_o \csc \beta d$$

If d is chosen small enough with respect to a wavelength at the operating frequency f, then

$$\tan \frac{\beta d}{2} \approx \frac{\beta d}{2}$$

Therefore,

$$Z_a \approx j R_o \frac{\beta d}{2} = j(L/C)^{1/2} \omega(LC)^{1/2} \frac{d}{2} = j \omega L \frac{d}{2}$$

Similarly,

$$Z_b \approx -j \frac{1}{\omega C d}$$

The Tee-network representing this short line is shown in Figure 4, where L is in henries per meter and C is in farads per meter.

The thin-film inductance can be made distributed just as the inductance
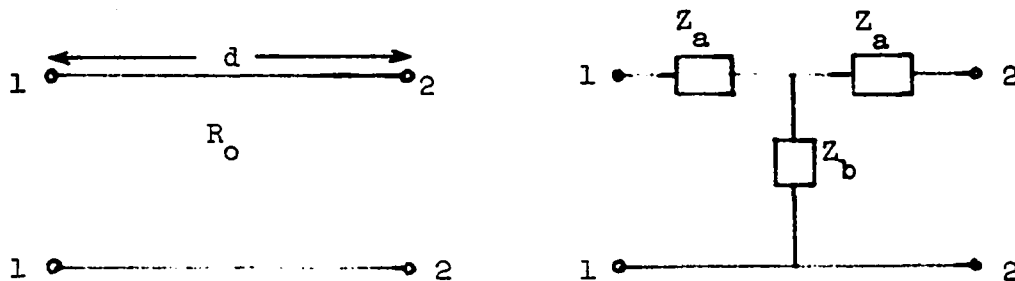
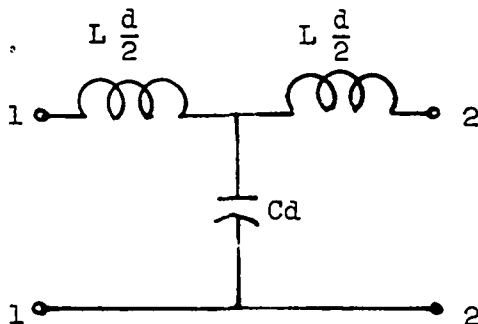Figure 3. Circuit representation of a transmission line

Figure 4. Tee equivalent circuit of an electrically short
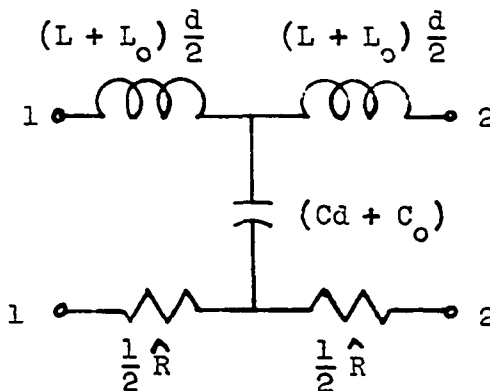transmission line

Figure 5. Tee equivalent circuit of an electrically short
line plated with thin film

of any general waveguide. If the thin film is plated or evaporated on a transmission line, its magnetization vector $\overline{M}$ will induce a voltage across the line. It should be kept in mind that the magnetic field excited by the pump current and that of the signal current are mutually orthogonal. The distributed inductance is a time-varying function and has the dimension of henries per meter. Due to the mixing action of f_s and f_p in the time-varying inductance, many frequencies are generated. By proper filtering or tuning it is possible to support one of these new frequencies, namely the idler frequency f_i such that Equation 35 is satisfied. Negative distributed resistance and distributed reactance will therefore exist. A short length of a transmission line of length d plated with thin film has the following parameters, with reference to Equations 33 and 34:

$$\hat{R}_s = -\frac{1}{2} L_y d\omega_s \frac{I_i}{I_s} \quad \text{ohms} \tag{37}$$

$$\hat{R}_i = -\frac{1}{2} L_y d\omega_i \frac{I_s}{I_i} \quad \text{ohms}$$

$$\hat{X}_s = L_o d\omega_s \quad \text{ohms} \tag{38}$$

$$\hat{X}_i = L_o d\omega_i \quad \text{ohms}$$

A small segment of a transmission line plated with thin film, has an equivalent circuit as shown in Figure 5. The added term C_o in farads will be discussed later.

Let

$$(L + L_o) \frac{d}{2} = L_t \frac{d}{2} \quad \text{henries}$$

and

$$C_d + C_o = C_t \quad \text{farads.}$$

It was mentioned above that the pump and signal fields must be at right angles with respect to each other. In a transmission line, the magnetic field due to the signal current is always in the transverse direction. Therefore, the pump field must be in the longitudinal direction. To achieve this requirement, a helix is wound around the conductor on which the film is plated. It is sufficient to have the film placed on one conductor in order not to increase the complexity of the construction. The helical propagation is a slow-wave propagation. In order to maintain the same phase relationship between the signal and the pump, they should travel with the same velocity. To slow the signal wave, a capacitance of C_o farads is connected across each segment of length d. Connecting the capacitors in this fashion is periodic in nature. Any such periodic structure exhibits stop and pass bands as shown by Watkins (17). Such bands are needed so that the signal and the idler frequencies are in the pass band while the other unwanted frequencies are the stop band.

The transmission line is now plated with film and a small segment is considered. The propagation constant can be found and a negative attenuation constant must exist if there is to be amplification.

Figure 6 shows the network used to find the propagation constant where Z_I is the image impedance. From circuit theory it can be shown that

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

where $\gamma = \alpha + j\beta$ is the propagation constant

α = the attenuation constant in nepers

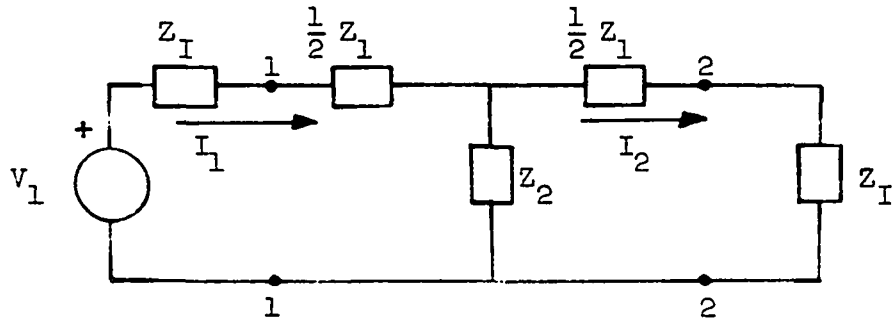β = the phase shift constant in radians.

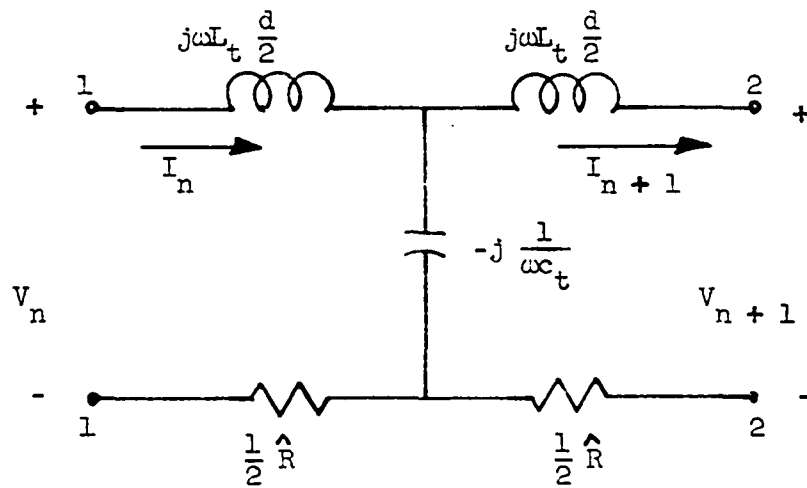Figure 6. Network used to compute the propagation constant

Figure 7. The equivalent circuit of the nth segment of the plate transmission line

In transmission lines where the parameters are distributed α is defined as nepers per meter and β is radians per meter and therefore γ has units of meter^{-1}. Applying this to the equivalent circuit of the nth segment of the plated transmission line as shown in Figure 7, the following is obtained:

$$\cosh \gamma d = 1 + \frac{\frac{1}{2}\hat{R} + j\omega L_t \frac{d}{2}}{-j\frac{1}{\omega C_t}}$$

$$= 1 - \omega^2 L_t C_t \frac{d}{2} + j \frac{1}{2}\hat{R}\omega C_t \qquad (39)$$

where ω is either ω_i or ω_s. Cosh γd can be expanded in terms of real and imaginary values.

$$\cosh \gamma d = \cosh \alpha d \cos \beta d + j \sinh \alpha d \sin \beta d \qquad (40)$$

Equating the real and the imaginary quantities in Equations 39 and 40 yields

$$\cosh \alpha d \cos \beta d = 1 - \omega^2 L_t C_t \frac{d}{2} \qquad (41)$$

$$\sinh \alpha d \sin \beta d = \frac{1}{2}\hat{R}\omega C_t \qquad (42)$$

Let it be assumed that the attenuation constant α is a small number, and d has been chosen to be small. Therefore, αd is a small number and

$$\cosh \alpha d \approx 1$$

$$\sinh \alpha d \approx \alpha d$$

Equations 41 and 42 now become

$$\cos \beta d = 1 - \omega^2 L_t C_t \frac{d}{2} \qquad (43)$$

$$\alpha \; d = \frac{\hat{R} \; \omega \; C_t}{2 \; \sin \beta d} \tag{44}$$

Equation 43 shows that for fixed L_t, C_t and d, there exist frequencies such that $\omega^2 L_t C_t \frac{d}{2}$ is greater than 2. This is true only if β is an imaginary number ($j\beta'$). Therefore

$$\gamma = \alpha + j\beta = \alpha - \beta' = \text{real number.}$$

This implies that the propagation constant γ is all attenuation with no propagation at all. Such frequencies are in the stop band of the amplifier. This condition must be satisfied so that all unwanted harmonics are beyond the cut-off frequency, otherwise the pump energy is used up in generating such harmonics. Although the transmission line supports the TEM mode with zero cut-off frequency, the line must be periodically loaded with capacitors to give definite stop and pass bands as explained by Watkins (17).

Equation 44 expresses the attenuation constant α in terms of the negative resistance $\hat{R}$. Therefore α is a negative attenuation constant. To be more specific, let the value of $\hat{R}$ at the two different frequencies given by Equation 37 be substituted in Equation 44, and

$$\alpha_s \; d = - \frac{L_y \; d \; \omega_s^2 \; C_t}{4 \; \sin \beta_s \; d} \left(\frac{I_i}{I_s} \right) \tag{45}$$

Also,

$$\alpha_i d = - \frac{L_y \; d \; \omega_i^2}{4 \; \sin \beta_i \; d} \left(\frac{I_s}{I_i} \right) \tag{46}$$

Bell and Wade (9) have shown that $\alpha_s = \alpha_i$. When this is substituted

in Equations 45 and 46 and both are combined, the general expression of α is obtained, namely:

$$\alpha^2 = \frac{(L_y\, C_t\, \omega_s\, \omega_i)^2}{16\,\sin\beta_s\, d\,\sin\beta_i\, d}$$

$$\alpha = \pm\, \frac{L_y\, C_t\, \omega_s\, \omega_i}{4(\sin\beta_s\, d\,\sin\beta_i\, d)^{1/2}} \tag{47}$$

This result indicates that both positive and negative attenuation constants exist along the propagating structure, which means that there will be two waves traveling along the guide with one wave being amplified and the other being attenuated.

After finding the attenuation and the gain constants in Equation 47, the currents flowing in the nth section of the line may be written as the superposition of two waves; one is growing and the other is decaying. They are

$$I_{sn} = e^{-j\beta_s dn} \left[A_1 e^{\alpha dn} + A_2 e^{-\alpha dn} \right] \tag{48}$$

$$I_{in} = e^{-j\beta_i dn} \left[A_3 e^{\alpha dn} + A_4 e^{-\alpha dn} \right] \tag{49}$$

where I_{sn} is the current in the nth section at the signal frequency and I_{in} is at the idler frequency. To have the complete picture of these currents, the constants A_1, A_2, A_3 and A_4 must be evaluated. Bell and Wade (9) have obtained similar equations for the voltages across the nth diode along the line, and also they have shown that

$$\frac{A_1}{A_3} = \frac{1}{j} \left(\frac{\sin\beta_i\, d}{\sin\beta_s\, d} \right)^{1/2} \frac{\omega_s}{\omega_i}$$

$$\frac{A_2}{A_4} = -\frac{1}{j} \left(\frac{\sin \beta_i \, d}{\sin \beta_s \, d} \right)^{1/2} \frac{\omega_s}{\omega_i}$$

or

$$A_3 = j \, A_1 \left(\frac{\sin \beta_s \, d}{\sin \beta_i \, d} \right)^{1/2} \frac{\omega_i}{\omega_s}$$

$$A_4 = -j \, A_2 \left(\frac{\sin \beta_s \, d}{\sin \beta_i \, d} \right)^{1/2} \frac{\omega_i}{\omega_s}$$

Using these results in Equations 48 and 49, the following is obtained:

$$I_{sn} = e^{-j\beta_s dn} \left[A_1 e^{\alpha dn} + A_2 e^{-\alpha dn} \right] \tag{50}$$

$$I_{in} = e^{-j(\beta_i dn - \frac{\pi}{2})} \left(\frac{\sin \beta_s \, d}{\sin \beta_i \, d} \right)^{1/2} \frac{\omega_i}{\omega_s} \left[A_1 e^{\alpha dn} - A_2 e^{-\alpha dn} \right] \tag{51}$$

The constants A_3 and A_4 have been expressed in terms of A_1 and A_2. To evaluate A_1 and A_2, the boundary conditions at the beginning of the line can be utilized. At $n = 0$

$$I_s = I_o$$
$$I_i = 0$$

Applying these to Equations 50 and 51, it can be seen that

$$A_1 = A_2 = \frac{I_o}{2}$$

Therefore

$$I_{sn} = I_o \, e^{-j\beta_s dn} \cosh \alpha \, dn \tag{52}$$

and

$$I_{in} = I_o e^{-j(\beta dn - \frac{\pi}{2})} \left(\frac{\sin \beta_s d}{\sin \beta_i d} \right)^{1/2} \frac{\omega_i}{\omega_s} \sinh \alpha \, dn \qquad (53)$$

The next step in finding the power gain is to find the voltage across the nth segment at both frequencies. This can be found by using Equations 52 and 53 and Figure 7.

$$V_{sn} = [\frac{1}{2} \hat{R}_s + j(\omega_s L_t \frac{d}{2} - \frac{1}{\omega_s C_t})] I_{sn} - j \frac{1}{\omega_s C_t} I_{s(n+1)} \qquad (54)$$

From transmission line theory, the current at any point with respect to the current a distance d away is given as

$$I_{s(n+1)} = e^{-\gamma d} I_{sn}$$

Therefore, Equation 54 now becomes

$$V_{sn} = [\frac{1}{2} \hat{R}_s + j(\omega_s L_t \frac{d}{2} - \frac{1 - e^{-\gamma d}}{\omega_s C_t})] I_{sn}$$

or

$$V_{sn} = Z_s I_{sn} \qquad (55)$$

where

$$Z_s = \frac{1}{2} \hat{R}_s + j [\omega_s L_t \frac{d}{2} - \frac{1 - e^{-\gamma d}}{\omega_s C_t}]$$

$$= \frac{1}{2} \hat{R}_s + j [\omega_s L_t \frac{d}{2} - \frac{1 - e^{-\alpha d}(\cos \beta_s d + j \sin \beta_s d)}{\omega_s C_t}]$$

Remembering that αd is a small number, it is implied that $e^{-\alpha d} \approx 1 - \alpha d$. Thus the expression for Z_s can be written as

$$Z_s = \frac{1}{2} \hat{R}_s + \frac{(1-\alpha d) \sin \beta_s d}{\omega_s C_t} + j[\omega_s L_t \frac{d}{2} - \frac{1 - (1-\alpha d) \cos \beta_s d}{\omega_s C_t}] \qquad (56)$$

From Equation 44

$$\frac{1}{2} \hat{R}_s = \frac{\alpha \, d \, \sin \beta_s d}{\omega_s C_t}$$

and from Equation 43

$$\omega_s L_t \frac{d}{2} = \frac{1 - \cos \beta_s d}{\omega_s C_t}$$

Substituting these expressions in Equation 56 yields

$$Z_s = \frac{\sin \beta_s d}{\omega_s C_t} + j \, \frac{\alpha d \cos \beta_s d}{\omega_s C_t}$$

The imaginary part is very small since it is multiplied by a small number αd. Therefore,

$$Z_s = \frac{\sin \beta_s d}{\omega_s C_t} \tag{57}$$

and

$$Z_i = \frac{\sin \beta_i d}{\omega_i C_t} \tag{58}$$

Substituting Equation 57 into Equation 55, the voltage across the nth section at the signal frequency becomes

$$V_{sn} = \frac{\sin \beta_s d}{\omega_s C_t} \, I_{sn}$$

$$V_{sn} = Z_s \, I_o \, e^{-j\beta_s dn} \cosh \alpha dn$$

By a similar argument

$$V_{in} = Z_i \, I_o \left(\frac{\sin \beta_s d}{\sin \beta_i d} \right)^{1/2} \frac{\omega_i}{\omega_s} \, e^{-j(\beta_i dn - \frac{\pi}{2})} \sinh \alpha dn$$

These phasor expressions may be transformed back to the time domain by using the fact that e^{j0} transforms into $\cos \omega t$. Therefore

$$v_{sn}(t) = Z_s I_1 \cos(\omega_1 t - \beta_s dn) \cosh \alpha dn \qquad (59)$$

$$i_{sn}(t) = I_o \cos(\omega_1 t - \beta_s dn) \cosh \alpha dn$$

Similarly

$$v_{in}(t) = Z_i \, I_o \left(\frac{\sin \beta_s d}{\sin \beta_i d}\right)^{1/2} \frac{\omega_i}{\omega_s} \sin(\omega_s t + \beta_i dn) \sinh \alpha dn \qquad (60)$$

$$i_{in}(t) = I_o \left(\frac{\sin \beta_s d}{\sin \beta_i d}\right)^{1/2} \frac{\omega_i}{\omega_s} \sin(\omega_s t - \beta_i dn) \sinh \alpha dn$$

These results are similar to the expressions Tien (18) has worked out, if in Equations 59 and 60, it is assumed that the phase shift per section is small such that $\sin \beta d \approx \beta d$. Tien has proposed to couple two transmission lines by a nonlinear inductance, and he approached his problem from transmission line theory using distribution parameters.

Now it is easy to find the power gain of this amplifier after finding expressions for the current and the voltage at any small segment. At the nth section

$$
\begin{aligned}
\text{power gain} &= \frac{P_{out}}{P_{in}} \\[6pt]
&= \frac{\frac{1}{2} \text{Re}[V_{sn} I_{sn}^*]}{\frac{1}{2} \text{Re}[V_o \, I_o^*]} \\[6pt]
&= \frac{Z_s \, I_o^2 \cosh^2 \alpha dn}{Z_s \, I_o^2} \\[6pt]
&= \cosh^2 \alpha dn \qquad (61)
\end{aligned}
$$

It must be shown next that these results of such a parametric device

satisfy the Manley-Rowe (1) energy relationships. To do this, the power gain at the two frequencies must be found and the ratio must be proportional to their frequency ratio.

$$P_s = \frac{1}{2} V_{sn} I^*_{sn} = \frac{1}{2} Z_s I_o{}^2 \cosh^2 \alpha dn$$

$$P_i = \frac{1}{2} V_{in} I^*_{in} = \frac{1}{2} Z_i I_o{}^2 \frac{\sin \beta_s d}{\sin \beta_i d} \sinh^2 \alpha dn$$

Therefore,

$$\frac{P_s}{P_i} = \frac{Z_s}{Z_i} \frac{\sin \beta_i d}{\sin \beta_s d} \frac{\omega_s{}^2}{\omega_i{}^2} \frac{\cosh^2 \alpha dn}{\sinh^2 \alpha dn} \qquad (62)$$

To have any reasonable value of gain n must be a large number. Since the hyperbolic sine and cosine of a large argument are about equal, then $\cosh^2 \alpha dn \approx \sinh^2 \alpha dn$. Therefore

$$\frac{P_s}{P_i} = \frac{Z_s}{Z_i} \frac{\sin \beta_i d}{\sin \beta_s d} \frac{\omega_s{}^2}{\omega_i{}^2} \qquad (63)$$

From Equations 57 and 58, the values of Z_s and Z_i are substituted in Equation 63 giving

$$\frac{P_s}{P_i} = \frac{\omega_s}{\omega_i}$$

EXPERIMENTAL WORK

To design a prototype model the first thing one must choose is the type of a transmission line, coaxial, open or strip type. The frequency must be chosen below the ferromagnetic resonance. All assumptions and approximations made throughout the analysis must be taken into account. Before proceeding in calculations, the design equations will be first summarized to see the big picture. The end result is obtaining gain as indicated by Equation 61, namely, $\cosh^2 \alpha dn$.

Step 1: Calculate L_o, the constant inductance given by Equation 25,

$$L_o = \frac{k \lambda_m}{H_k + H_b}$$

where k depends on the configuration of the transmission line and H_k depends on the film, and H_b is the d-c bias field.

Step 2: Choose a as a small number such that the approximation of Equation 28 is valid. L_y is then readily found.

Step 3: Choose the section length d and also C_o, such that Step 4 gives real phase shift constants; thus propagation exists.

Step 4: Calculate the phase shifts at the two frequencies by using Equation 43:

$$\cos \beta d = 1 - \frac{1}{2} \omega^2 C_t L_t d$$

Step 5: Calculate α as given by Equation 47:

$$\alpha = \frac{L_y C_t \omega_s \omega_i}{4(\sin \beta_s d \sin \beta_i d)^{1/2}}$$

Step 6: Choose a number n large enough to get a reasonable gain and such that the assumption in Equation 62 is valid. That is $\cosh^2 \alpha dn$ $\approx \sinh^2 \alpha dn$. The gain is readily found using Equation 61.

Step 7: All assumptions and approximations must be checked for validity:

a) Is d small enough that $\tan \frac{\beta d}{2} \approx \frac{\beta d}{2}$?

b) Is αd small enough such that the assumptions made to obtain Equations 43 and 44 are valid? Namely,

$$\cosh \alpha d \approx 1$$

$$\sinh \alpha d \approx \alpha d$$

c) In Equation 62, n was said to be large such that $\cosh^2 \alpha dn \approx \sinh^2 \alpha dn$. Is this true?

d) Is the velocity of propagation in the signal line equal to that of the pump line? In the signal line

$$v = \frac{1}{\left(L_t \, C_t \right)^{1/2}}$$

and in the pump

$$v = \frac{\omega_p}{\beta_p}$$

where

$$\beta_p = \beta_s + \beta_i$$

Step 8: Calculate the pump field using Equation 27

$$H_p = a(H_k + H_b) \text{ Amp-turns per meter}$$

The number of turns depends on the pitch of the helix. Therefore,

the pump current can be found. The r-f pump power can be calculated if the characteristic impedance of the helix is known.

The above steps tell the whole story of designing the traveling-wave parametric amplifier. It may be noticed that there are many variables in such a device. The choice of such variables depends primarily on experience.

A prototype model was constructed to operate at signal frequency of 80 Mc and pump frequency of 200 Mc. The choice of the signal was that it must be far below the ferromagnetic resonance, while the choice of the pump was mainly due to the availability of the equipment in the laboratory. It followed then that the idler frequency was 120 Mc. The transmission line was a strip type of line. It is impossible, so far, to plate or evaporate thin film on metal strip (such as copper), because the surface cannot be made smooth enough so that no domains will form perpendicular to the surface. This is the reason for not choosing either a coaxial line or an open line. The film was evaporated on 6-mil glass substrate. The substrates were then sliced 3 mm wide and fastened to one conductor of the strip line. To continue the description of this model, the above steps will be followed:

Step 1: k was defined as the proportionality constant in $H = ki$. The magnetic field just outside a thin sheet of width w and thickness of t carrying current i is given as

$$H = \frac{i}{2w}$$

Therefore

$$k = \frac{1}{2w} .$$

The flux linkage λ_m = flux x number of turns.

flux = (B + M) x cross sectional area of the film

M > > B and M = 1 weber per meter^2

area = t x 1 = t meter^2

λ_m = 2(Mt) for both sides of the conductor

Therefore

$$L_o = \frac{t}{w(H_k + H_b)} \text{ henry per meter}$$

t = 3000 A^o

w = 3 mm

H_k = 200 Amp-turns per meter as measured in the laboratory

Let H_b = 200 Amp-turns per meter.

It should be noted that there is no way in the laboratory to measure thickness exactly.

$$L_o = 3.75 \times 10^{-7} \text{ henry per meter.}$$

The distributed inductance was negligible with respect to L_o. Therefore,

$$L_t = 3.75 \times 10^{-7} \text{ henry per meter.}$$

Step 2: Let a = 0.4

$$L_y = a L_o = 1.5 \times 10^{-7} \text{ henry per meter.}$$

Step 3: Let d = 0.5 cm

and C_o = 300 x 10^{-12} farad per section

The distribution capacitance was negligible with respect to C_o. Therefore,

$$C_t = C_o = 300 \times 10^{-12} \text{ farad per section.}$$

Step 4: $\cos \beta_s d = 1 - \frac{1}{2} \omega_s^2 C_t L_t d$

$= 1 - \frac{1}{2}(5.02 \times 10^8)^2(300 \times 10^{-12})(1.5 \times 10^{-7})(\frac{1}{2} \times 10^{-2})$

$= 0.929$

$\cos \beta_i d = 1 - \frac{1}{2}(7.54 \times 10^8)^2(300 \times 10^{-12})(1.5 \times 10^{-7})(\frac{1}{2} \times 10^{-2})$

$= 0.84$

$\beta_s d = 0.384$ and $\sin \beta_s d = 0.375$

$\beta_i d = 0.575$ and $\sin \beta_i d = 0.545$

Step 5: $\alpha = \dfrac{L_y C_t \omega_s \omega_i}{4(\sin \beta_s d \sin \beta_i d)^{1/2}}$

$= \dfrac{(1.5 \times 10^{-7})(300 \times 10^{-12})(5.02 \times 10^8)(7.54 \times 10^8)}{4(.375 \times .545)^{1/2}}$

$= 9.5$ nepers per meter

Step 6:   Let $n = 46$

$\alpha dn = (9.4)(\frac{1}{2} \times 10^{-2})(46) = 2.16$

$\cosh \alpha dn = 4.39$

gain $= \cosh^2 \alpha dn = 19.3 = 12.8$ db

Step 7:   The validity of the approximations will now be tested.

a) $\beta_s d = 0.384$

$\beta_i d = 0.575$

$\tan \dfrac{\beta_s d}{2} = 0.194$ and $\dfrac{\beta_s d}{2} = 0.192$

$\tan \dfrac{\beta_i d}{2} = 0.292$ and $\dfrac{\beta_i d}{2} = 0.287$

b) $\alpha d = (9.4)(\frac{1}{2} \times 10^{-2}) = 0.047$

$\cosh 0.047 = 1.001 \approx 1.0$

$\sinh 0.047 = 0.047$

c) $\cosh^2 \alpha dn = 19.3$

$\sinh^2 \alpha dn = 18.3$

$\dfrac{\cosh^2 \alpha dn}{\sinh^2 \alpha dn} = 1.05 \approx 1.0$

d) $v = \dfrac{1}{\left(L_t \, C_t\right)^{1/2}}$

$C_t = 300 \times 10^{-12}$ farad per section

$\quad = (300 \times 10^{-12})(200 \text{ sections per meter})$

$\quad = 6 \times 10^{-8}$ farad per meter

$L_t = 3.75 \times 10^{-7}$ henry per meter

$v = 6.67 \times 10^6$ meters per second

$\beta_p = \beta_s + \beta_i = \dfrac{.959}{0.5 \times 10^{-2}} = 191.8$ radians per meter

$v = \dfrac{\omega_p}{\beta_p}$

$v = \dfrac{1.256 \times 10^9}{191.8} = 6.6 \times 10^6$ meters per second

The cut-off frequency of the amplifier calculated using Equation 43 is 425 MC. This implies that some harmonics of the signal and idler frequencies will exist. However, at higher frequencies the resistance of the thin film in Equation 25 will not be negligible. The line therefore will be lossy, and such harmonics attenuate.

To construct the pump line so that the wave travels with the

calculated velocity, the pitch angle of the helix can be esti-
mated by simple geometry. The actual angle will be far from the
calculated value. The reason is that inside the helix there
were the signal lines with many capacitors, a film, and other
supporting materials. It was easier just to wind a coil with
a known number of turns per unit length and see if both signal
and pump lines exhibit the same delay when a pulse was applied
at the input. Using this method, it was found that 300 turns
per meter satisfied the requirement and exhibited a 25-nano-
second delay.

Step 8: H_p = 0.4(400) = 160 Amp-turns per meter.

If the number of turns per meter is 300, then the current
should be 0.534 ampere.

The characteristic impedance of the pump line was measured
and not calculated. The input impedance was measured for dif-
ferent loading resistors. It was found to be 200 ohms. The
r-f power required to drive this line was 56.7 watts -- a very
large power to obtain in the present laboratory.

The prototype model was constructed according to the above specifica-
tions. A photograph, Figure 8, shows the actual model; its exploded cross
section is shown in Figure 9. The model was inserted in a set-up as shown
in Figure 10. The actual output power of the pump source was 7 watts.
Half of this power was lost in the triple-stub matching network. When 3.5
watts of power is applied to this system, it can be calculated, by going
backward through the above procedure, that the voltage gain is only 5%.

Figure 8a. The signal line

Figure 8b. The helical pump envelops the signal line

Figure 8c. The bias winding envelops both propagating
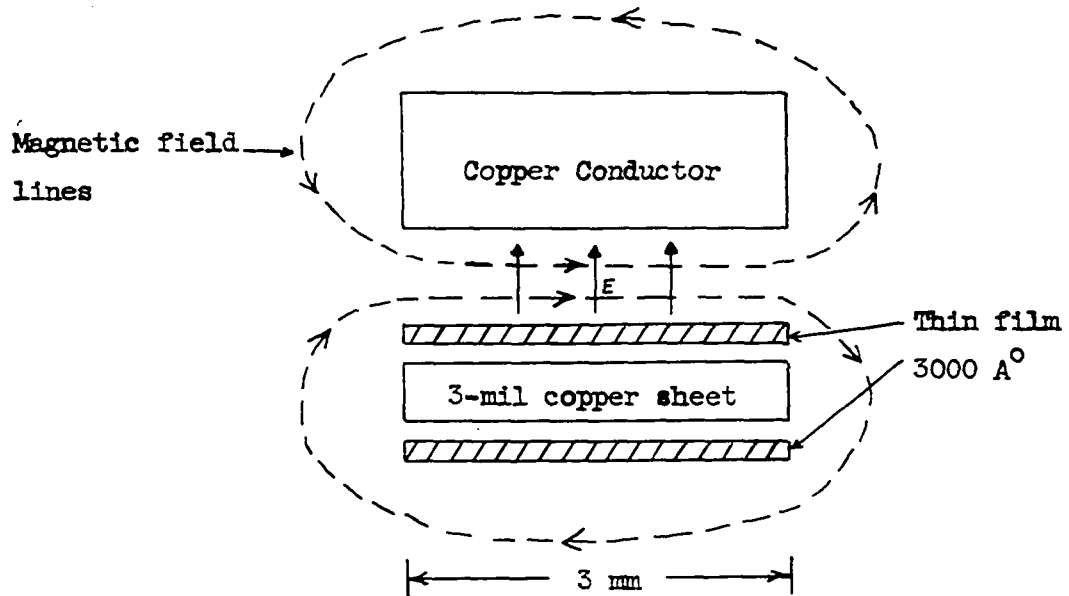   structures

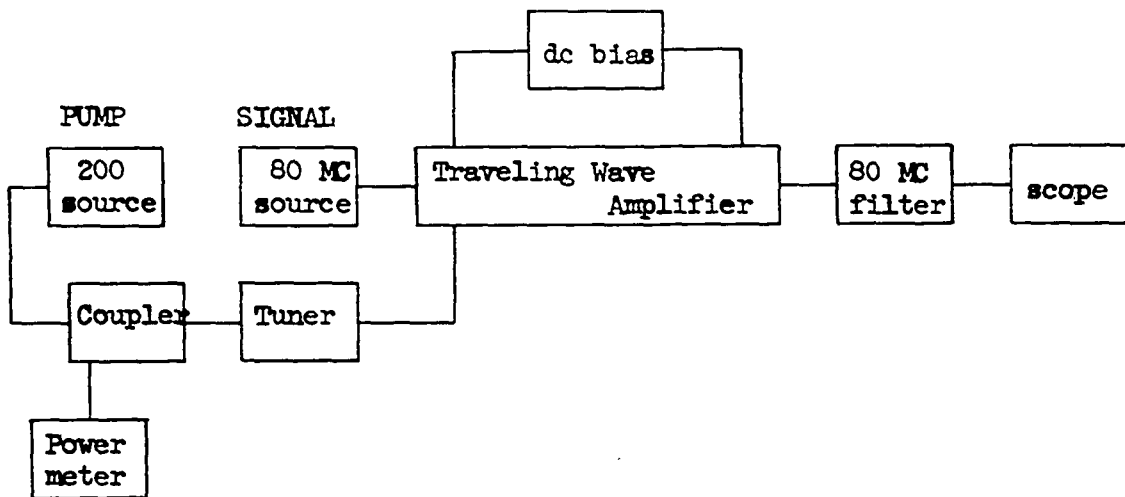Figure 9. Exploded cross sectional view of the line

Figure 10. Block diagram of the set-up used for the experiment

When the pump power was turned on, a 2-3% increase in the amplitude of the signal was observed on the face of the oscilloscope. This was an estimate since it is hard to measure such low values accurately. The pump power was not enough to give a net total gain for the line, but it did tend to reduce the attenuation in the signal circuit. As evidence that this increase was due to the presence of the film, and not a matter of coincidence, a similar line was previously tested with a negligibly thin layer of film (approximately 500 A°) deposited on the glass substrate. It should be noted that for practical purposes such a thin deposit has no effect. When the pump power was turned on, absolutely no change in the output was observed on the face of the oscilloscope. Therefore, it can be concluded that the above mentioned increase was due to the magnetic thin film. Thus some of the pump power was transferred into the signal circuit which supports the preceding theory on parametric amplification.

SUMMARY

This study presents the application of ferromagnetic thin film in traveling-wave parametric amplifiers. Due to the frequency mixing action in the film, negative resistance is introduced in the system, hence gain.

A ferromagnetic thin film deposited on a glass substrate is found to have a magnetization vector $\overline{M}$ in a preferred direction often called the easy direction, normal to which is called the hard direction. Two orthogonal coils are wound around this film. One is called the pump and its magnetic field is parallel to $\overline{M}$. The other is called the signal; its field is in the hard direction. If these fields are excited, energy is transferred from the pump to the signal due to the voltage induced in the signal circuit by the time rate of change of vector $\overline{M}$. This change is caused by the vibration of $\overline{M}$ as a result of the interaction of the two fields. The angles through which $\overline{M}$ vibrates are described by two equations derived from the Landau-Lifshitz equation. When the two equations are combined, the resulting equation is recognized to be analogous to that of a G, C, and L circuit, where the inductance is a function of time. The ferromagnetic resonant frequency of an 80 percent nickel, 20 percent iron permalloy thin film is about 600 Mc if the d-c field is 200 Amp-turns per meter. If the signal frequency is far below resonance, the only effective element in the circuit is the time-varying inductance.

Manley and Rowe (1) showed that if two signals are applied to a time-varying reactance, frequencies of the sums and differences of the harmonics of the applied signals are generated. Any harmonic can be taken out by proper filtering. If the sum of the two applied frequencies is equal to

the frequency at which the reactance varies, negative resistance is found to exist. This resulting element gives parametric amplification.

A traveling-wave parametric amplifier is a waveguide along which the wave travels and is also amplified due to the negative resistance. The analysis of the problem was carried out from a circuit theory view point by considering the line to be many small sections in cascade. Each section was represented by a two-port network. The attenuation constant α was found to have positive and negative components. The voltage and current wave equations were developed as functions of α and β, the propagation constant. Power gain was then readily found. It was also proved that the powers and frequencies involved in this device satisfied the Manley-Rowe (1) general energy equations.

A traveling-wave amplifier was designed and constructed at a signal frequency of 80 Mc and a pump frequency of 200 Mc. The signal traveled on a strip transmission line with one conductor covered with a thin film of permalloy. The pump power was carried over a helix wound around the signal line. A 300 pf capacitor was connected across each 0.5-centimeter-long section. The capacitors were introduced to slow the signal wave down because the helical propagation of the pump power was slow, and to exhibit stop and pass bands in the amplifier. The synchronism between the pump and signal was essential so that the phase relationship between both stayed constant. It was found that to obtain 13 db gain more than 50 watts of r-f power was needed for the pump, which was far more than was available. However, when the power available was applied, a slight increase in the amplitude of signal was observed. The pump power was not enough to

give a net total gain for the line, but it did tend to reduce the attenuation in the signal circuit. Thus some of the pump power was transferred into the signal circuit as predicted by the parametric amplification theory.

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